у цілому добре відтворюється у моделюванні методом кластерної динаміки як для вольфраму без домішків, так і для вольфраму з вуглецем. Експериментальні піки відновлення на другої стадії відновлення дефектної структури вольфраму формуються, згідно з нашим дослідженням, завдяки взаємодії точкових дефектів та атомів вуглецю.

Ключові слова: відпал, опромінювання, вольфрам, вуглець, кластерна динаміка.

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BEHAVIOR OF MATERIAL WITH A MEMORY OF FORM AND PSEUDOELASTICITY UNDER NONSTATIONARY LOADING OF THE BODY

A nonstationary thermo-elastic-plastic problem is examined for pseudoelastic bodies. The key feature of theory consists in that the diagram of tension of deformations appears as a three-unit broken line and can have a falling down segment. Thus the characteristic points of the diagram depend on the material's temperature and phase state. Such character of the diagram leads to the discontinuous *solutions and as a result to the moving boundaries of phase transitions. The example of thin stripe at uniaxonic tension is considered. It is shown that deformation is not homogeneous through the stripe and its development depends on the material's properties. The got results confirm an idea that front of races change of deformation spreads with permanent speed that depends only on mechanical properties of material.*

Keywords: thermo-elastic-plasticity, pseudoelasticity, form memory, phase transitions.

1. Introduction

The list of alloys that exhibit pseudoelasticity includes Ni-Ti alloys and various copper, iron, silver and gold-based alloys. Pseudoelasticity is the ability of a material to accumulate deformations upon loading at a high temperature regime and then return to its original state after unloading (through the hysteresis loop). The mechanism of this reduction is the transformation from the martensite phase to the original austenite phase.

Such alloys as NiTi, CuZnAl, CuAlNi, AuCd and others can restore deformations up to 3%. Important characteristics of some of these materials are internal damping, pseudoelasticity and high yield strength. It is noted that the amount of experimental data of high quality of macroscopic behavior of NiTi remains limited.

A characteristic feature of the SPF material diagram under active loading is an area of ideal plasticity (NiTi stress-strain response at 70 ° C [3]). Similar sections are also present at unloading, but at certain temperatures.

2. Local diagram of pseudelastic material

To describe the local relationship between physical quantities, a model of an elastoplastic body with a softening point under active loading and constant temperature was used [1-3]. The temperature field of the body is assumed to be known, being obtained by solving the corresponding problem of nonstationary thermal conductivity or from other sources [2].

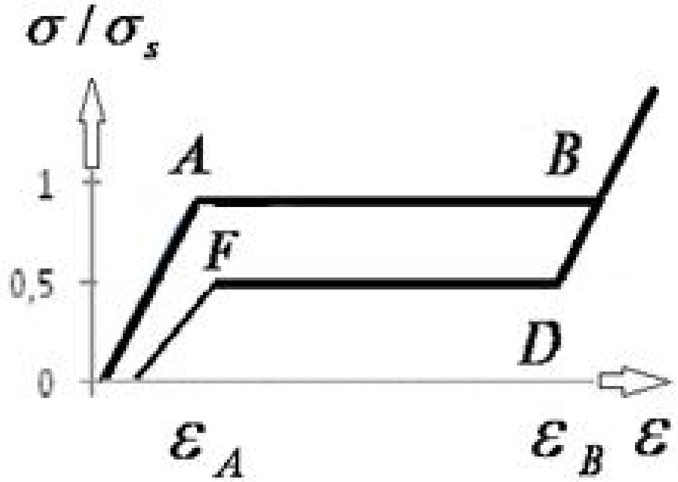

Fig. 1. The local material diagram.

The table below gives the coordinates of the points is obtained on the basis of the processing of the experimental data given in [3].

T^0C	$A(\varepsilon\%, \sigma\,(GPa))$	$B(\varepsilon\%, \sigma\,(GPa))$	$D(\varepsilon\%, \sigma\,(GPa))$	$F(\varepsilon\%, \sigma\,(GPa))$
100	1,00; 0,82	6,50; 0,82	6,05; 0,45	0,55; 0,45
90	1,00; 0,78	6,50; 0,78	6,03; 0,41	0,53; 0,41
80	1,00; 0,67	6,50; 0,67	6,07; 0,38	0,57; 0,38
70	1,00; 0,59	6,30; 0,59	5,81; 0,30	0,51; 0,30
60	1,00; 0,44	6,20; 0,44	5,72; 0,23	0,52; 0,23
50	1,00; 0,42	5,80; 0,42	5,16; 0,15	0,36; 0,15
40	1,00; 0,39	5,70; 0,39	4,88; 0,07	0,18; 0,07
30	1,00; 0,31	5,00; 0,31	4,00; 0,00	0,00; 0,00
20	1,20; 0,28	4,30; 0,28	3,10; 0,00	0,00; 0,00
10	1,40; 0,22	4,30; 0,22	2,90; 0,00	0,00; 0,00
0	2,00; 0,20	3,90; 0,20	1,90; 0,00	0,00; 0,00

When the temperature changes during the loading process, the transition from one diagram to another occurs. Different local diagrams of the material can be used at different points of the body.

3. Statement of the non-stationary problem of the theory of thermo-elastic-plasticity for a pseudoelastic material

Let us determine the velocity of the slow wave from which the plastic deformation field propagates along the one-dimensional body $x \in [0; L]$. On the edge $x = 0$ the speed of stretching the sample $v = V_0$ is set and its edge $x = L$ is fixed and here $v = 0$.

In general, the required values are: the speed of movement in the axial direction $v(x,t)$ (the movement $u(x,t)$ is determined if necessary by integration $v(x,t)$ over time); axial stress $\sigma(x,t)$; axial strain $\varepsilon(x,t)$ and temperature $T(x,t)$. Here $x \in [0; L], t \in [0, \infty)$.

To determine the unknown quantities, we use a system of equations

$$\rho \cdot \frac{\partial v}{\partial t} = \frac{\partial \sigma}{\partial x},\ \dot{\varepsilon} = \frac{\partial \varepsilon}{\partial t} = \frac{\partial v}{\partial x},\ \sigma = \begin{cases} E_1 \varepsilon - K\alpha_T (T - T_0) & npu\ \ \varepsilon \in [0, \varepsilon_S], \\ E_2(\varepsilon - \varepsilon_S) + \sigma_S - K\alpha_T (T - T_0) & npu\ \ \varepsilon \in [\varepsilon_S, \varepsilon_C], \\ E_3(\varepsilon - \varepsilon_c) + \sigma_C - K\alpha_T (T - T_0) & npu\ \ \varepsilon \in (\varepsilon_C, \infty), \end{cases} \quad (1)$$

$$\frac{\partial T}{\partial t} = a^2 \frac{\partial^2 T}{\partial x^2} + W.$$

Here, $\rho -$ is the density of the material, E_1, E_2, E_3 is the modules of the local material diagram (Figure 1), as well as the coefficient of linear thermal expansion α_T, which can depend on temperature, $W -$ source of heat, which is released as a result of a phase transition.

We pass in system (1) to dimensionless normalized quantities for which we retain the previous notation

$$v \Rightarrow \frac{v}{v_*}, \varepsilon \Rightarrow \frac{\varepsilon}{\varepsilon_{ST}}, \sigma \Rightarrow \frac{\sigma}{\sigma_{ST}}, T \Rightarrow \frac{T}{T_*}, x \Rightarrow \frac{x}{x_*}, t \Rightarrow \frac{t}{t_*}. \quad (2)$$

Here v_*, T_*, x_*, t_* are some given scale values of displacement speed, temperature, spatial coordinate and time. $\sigma_{ST}, \varepsilon_{ST}\ (\sigma_{ST} = E_1(T_*)\varepsilon_{ST})$ The flow stresses of the material for stress and deformation, determined at the $T = T_*$.

As a result of the transition to dimensionless normalized quantities (2), we rewrite the system of equations (1) this way

$$\frac{\partial v}{\partial t} = k_{1*} \frac{\partial \sigma}{\partial x},\ \frac{\partial \varepsilon}{\partial t} = k_{2*} \frac{\partial v}{\partial x},$$

$$\sigma = \begin{cases} E_{1*}\varepsilon - K_* \alpha_{T*}(T - T_0) & npu\ \ \varepsilon \in [0; \varepsilon_S], \\ E_{2*}(\varepsilon - \varepsilon_S) + \sigma_S - K_* \alpha_{T*}(T - T_0) & npu\ \ \varepsilon \in [\varepsilon_S; \varepsilon_C], \\ E_{3*}(\varepsilon - \varepsilon_c) + \sigma_C - K_* \alpha_{T*}(T - T_0) & npu\ \ \varepsilon \in (\varepsilon_C; \infty). \end{cases} \quad (3)$$

The notation

$$k_{1*} = \frac{\sigma_{ST} t_*}{\rho x_* V_*},\ k_{2*} = \frac{v_* t_*}{x_* \varepsilon_{ST}}, E_{1*} = \frac{E_1(T)}{E_1(T_*)}, E_{2*} = \frac{E_2(T)}{E_2(T_*)}, E_{3*} = \frac{E_3(T)}{E_3(T_*)},$$
$$\alpha_{T*} = \frac{\alpha_T T_*}{\varepsilon_{ST}},\ K_* = \frac{E_{1*}}{1 - 2v},\ k_{3*} = t_* \frac{a^2}{x_*^2}. \quad (4)$$

To simplify the calculations, we choose $k_{2*} = 1$. Then we can take

$$v_* = \varepsilon_{ST}\frac{x_*}{t_*}, \qquad k_{1*} = \frac{E_1 t_*^2}{\rho x_*^2}$$

We use the finite-difference method. To numerically solve the system (3), we introduce grids with respect to time t and coordinate x as follows [4]

$$\omega_t = \left\{t_p; t_{p+1} = t_p + \tau; t_0 = 0; p = 0;1;2;...\right\},$$
$$\omega_h \left\{x_i; x_{i+1} = x_i + h; x_0 = 0;\ h = \frac{L}{n}; i = 0;1;2;...n\right\}. \tag{5}$$

Then an explicit difference system equivalent to a complete system of partial differential equations (3) can be written as

$$v^{p+1} = v^p + \tau k_{1*}\lambda(\sigma^p), \varepsilon^{p+1} = \varepsilon^p + \tau\lambda(v^p), \tag{6}$$
$$T^{p+1} = T^p + \tau k_{3*}\mu(T^p) + \tau W^p t_*.$$

We shall formulate the boundary conditions for the heat equation as the conditions for free heat exchange

$$\frac{\partial T}{\partial x} = 0; \quad x = 0; L.$$

From this we obtain the calculated difference formulas on the boundary of the rod

$$T_0^p = (k_2 T_1^p - k_3 T_2^p + k_4 T_3^p)/k_1\ ;\ T_n^p = (k_2 T_{n-1}^p - k_3 T_{n-2}^p + k_4 T_{n-3}^p)/k_1. \tag{7}$$

The magnitude of the stress at an arbitrary instant of time is determined explicitly by the corresponding formula in (3). In the calculation formulas (6) we introduce the notation for difference operators approximating the first derivatives with respect to the coordinate. They can be set in various ways.

Here, to approximate the derivatives, we obtain the following difference formulas [4]. Equations (7) use the coefficients determined by using the spline function. If cubic B-splines that have a fourth order of approximation are used, then

$$k_1 = 11;\ k_2 = 18;\ k_3 = 9;\ k_4 = 2.$$

In the case of using strained splines that have the fifth order of approximation $k_1 = 11,2646;\ k_2 = 18,4641;\ k_3 = 9,1344;\ k_4 = 1,9349.$

4. Results of numerical experiment

Let's consider a series of numerical results. In Fig. 2, the left column shows the time variation of the deformation and stress fields under active loading. The transition from A to B, austenite to martensite ($A \to M$) occurs at $V_0 = -1,3v_*$. The right column shows the deformation and stress field changes in time during the reverse transition ($M \to A$) from D to F at $V_0 = 1,7v_*$. Lines 1 give the distribution of deformations along the length of the rod at fixed instants of time, and lines 2 show the corresponding distribution of stresses in the body.

The obtained results support the assumption that the front of the stepwise change in strain propagates at a constant rate, which depends only on the mechanical properties of the material.

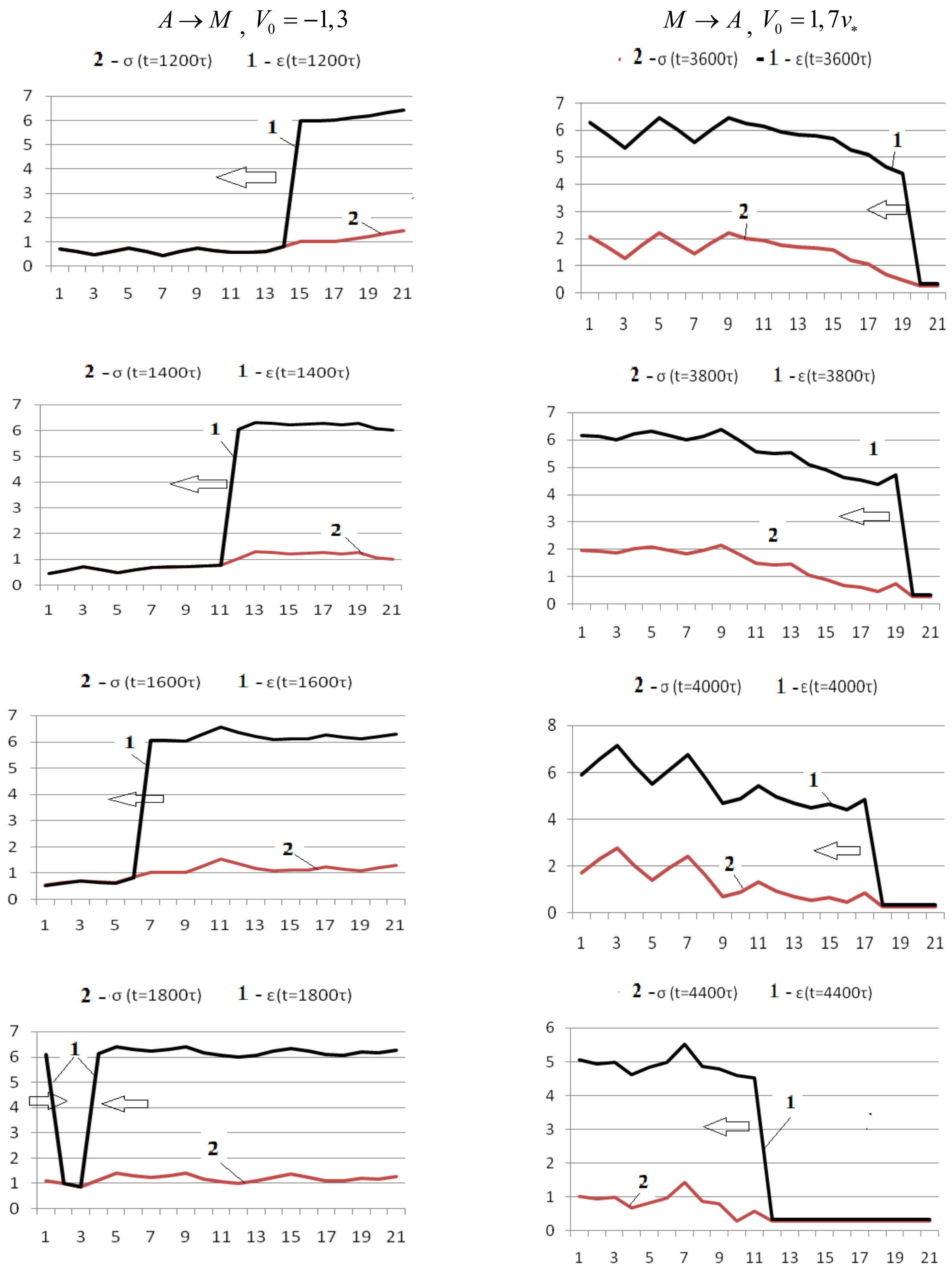

Fig. 2. Distribution of stresses and plastic deformations for different instants of time ($\tau = 0{,}001$).

The change in the temperature field along the axis of the rod for different instants of time is shown in Fig. 3.

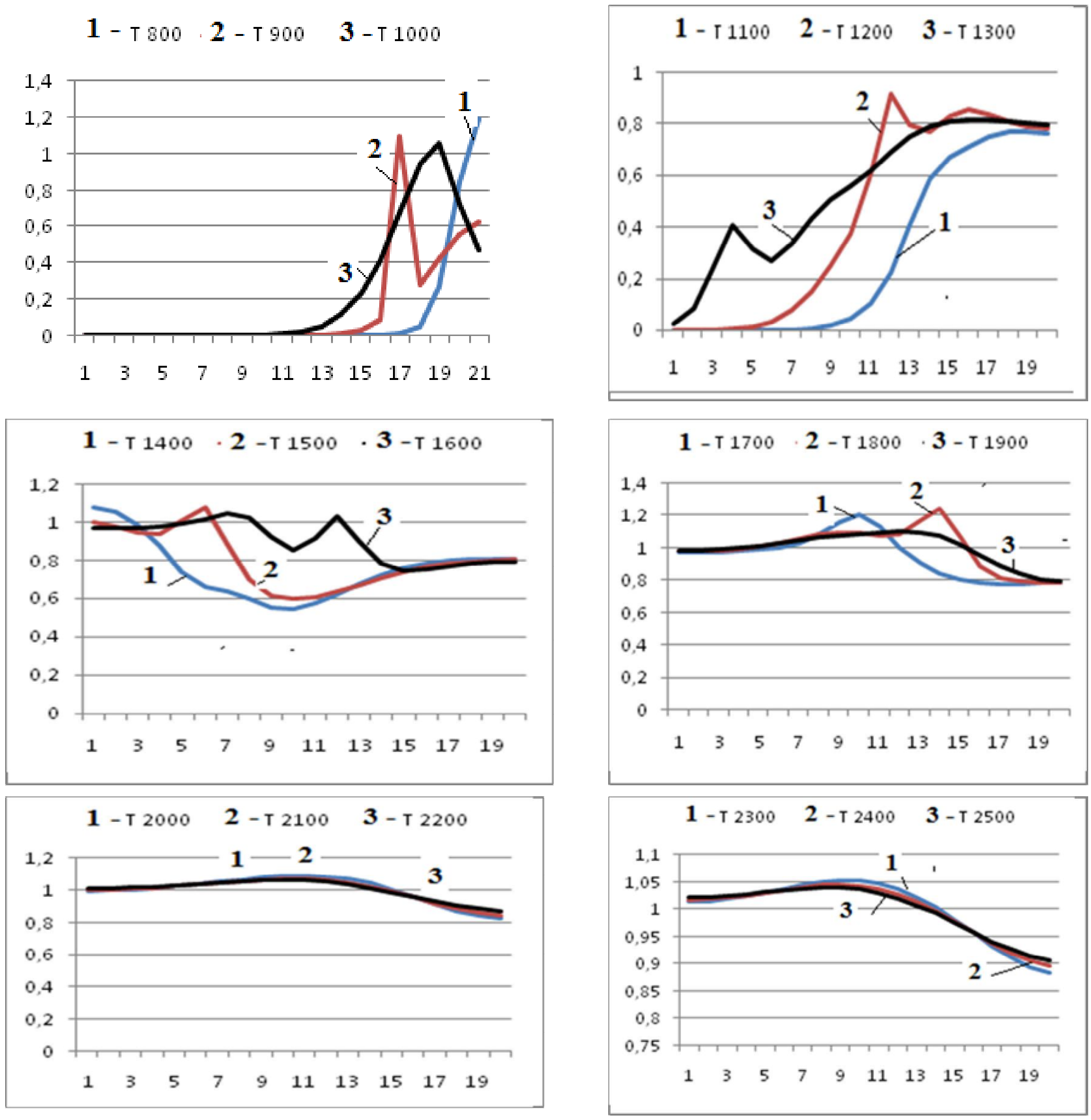

Fig. 3. The time variation in the temperature field due to heat release.

Here, the temperature field arises in connection with the stress-strain state and is due to heat generation during the sequence of phase transitions (jumps in the diagram from Composition A to B). The results are presented in dimensionless form, the temperature at the phase transition point is assumed to be equal to the conventional unit. Numbers 1, 2, 3 indicate the curves for the specified time.

5. Conclusions

A version of the model of behavior of a pseudoelastic material has been developed and experimentally substantiated. In this model, the possibility of quantitative evaluation of the associated interactions between stresses, temperature, deformation and material loading rate is built that is suitable for modeling the continual level.

We obtained the numerical confirmation that the front of the stepwise change in the strain propagates at a constant rate that depends only on the mechanical properties of the material.

Recurrent formulas allow us to obtain the third (for temperature) and fourth (for the velocities of displacements, stresses and deformations) order of approximation of the method with respect to the coordinates.

When constructing the solution of the complete system of thermomechanical equations, all unknown quantities were represented as spline functions [4]. This makes it possible to write more accurate difference expressions for the differential operators that make up the difference schemes and, on the whole, increase the accuracy of the computation by coordinates by at least an order of magnitude.

Given the same accuracy of calculations with the classical finite-difference method, this method allows us to obtain results faster by virtue of the choice of larger steps of integration along the coordinates, which leads to a significant decrease in the number of nodes of the spatial grid used.

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Анотація. ***Петров А. Д. Поведінка материалу з пам’яттю форми та псевдопружністю при нестаціонарному навантаженні тіл***. *Розглядається нестаціонарна термо-пружно-пластичне задача для тіл з пам’яттю форми. Особливість теорії полягає в тому, що діаграма напруги деформацій в матеріальній точці представляється у вигляді триланкової ламаної і може мати спадаючу ділянку. При цьому характерні точки діаграми залежать від температури і фазового стану матеріалу. Такий характер діаграми призводить до розривних рішень і як наслідок до рухливих меж фазових переходів. Розглянутий приклад тонкої смуги при одноосному розтягуванні. Отримані результати підтверджують думку про те, що фронт стрибкоподібної зміни деформації поширюється з постійною швидкістю, яка залежить лише від механічних властивостей матеріалу і температури.*

Ключові слова: термо-пружно-пластичність, псевдопружність, пам'ять форми, фазові переходи.

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