"worst" point is determined. 4. The "worst" points are sequentially removed as long as the correlation coefficient reaches the desired magnitude.

More over there have been analyzed the features of remote compounds that distinguish them from the main array.

Keywords: perovskites, ionic conductivity, descriptor, correlation, regression analysis, computer modeling.

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K.O. Buryachenko

SOLVABILITY OF THE NEUMANN PROBLEM FOR SOME CLASSES OF IMPROPERLY ELLIPTIC FOURTH ORDER EQUATIONS

There have been explored and established the sufficient conditions of solvability of the Neumann problem for one class of improperly elliptic fourth-order general equations in a disk K *in space* $C^4(K)\cap C^{3,\alpha}(\overline{K})$*.With the help of Chebyshev`s polynomials we build solutions of the Neumann problem.*

Key words: improperly elliptic equations, properly elliptic equations, fourth order partial differential equations, Neumann problem, Dirichlet problem, kernel, Chebyshev`s polynomials.

1. Introduction

This paper is devoted to the existence of a solution of the Neumann problem in a disk for fourth-order improperly elliptic differential equations of general form. The range of problems of our work belongs to fairly important questions of the correctness of the so-called general boundary-value problems for high-order differential equations, which spring from the works by Hormander and Wischik, who, with the help of extension theory, proved the existence of a correct boundary problem for linear differential equations of arbitrary order with constant complex coefficients in a bounded domain with a smooth boundary. For high-order equations, in particular, for fourth-order equations and later for equations of arbitrary even order 2m, $m>1$, the Dirichlet problem was studied by Babayan [1, 9] and Buryachenko [2, 4, 5]. As to the Neumann problem, some conditions of its solvability in a disk for second-order equations without the lowest terms were obtained in the recent work by Burskii and Lesina [3], and for equations containing the lowest part by Bonanno [10]. For equations of general form with a homogeneous symbol of the fourth order and higher, the Neumann problem has been investigated only for the properly elliptic equation (see [6]). In the present work, we propose to carry over the methods of investigations of our previous works [6,7] , which were applied for studying the properly elliptic Neumann problem for the case of improperly elliptic Neumann problem. In this work we construct solution of such problem with the help of Chebyshev`s polynomials.

2. Problem statement

Let us consider the Neumann problem in a disk for fourth-order improperly elliptic equations with constant complex coefficients:

$$L(\partial_x)u = a_0\frac{\partial^4 u}{\partial x_1^4} + a_1\frac{\partial^4 u}{\partial x_1^3\partial x_2} + a_2\frac{\partial^4 u}{\partial x_1^2\partial x_2^2} + a_3\frac{\partial^4 u}{\partial x_1\partial x_2^3} + a_4\frac{\partial^4 u}{\partial x_2^4} = 0, \quad (1)$$

$$\left.\frac{\partial^2 u}{\partial \overline{n}^2}\right|_{\partial K} = f_1, \ \left.\frac{\partial^3 u}{\partial \overline{n}^3}\right|_{\partial K} = f_2, \quad (2)$$

where $\overline{n}$ is the unit vector of an external normal, $\partial_x = \left(\frac{\partial}{\partial x_1}, \frac{\partial}{\partial x_2}\right)$, $a_k \in C, k = 0,...,4,$ $f_1 \in C^{1,\alpha}(\partial K)$ and $f_2 \in C^{\alpha}(\partial K)$, $0 < \alpha < 1$, are functions specified on the boundary ∂K which can be prolonged to analytic functions in the disk K and outside it. We recall the important definition, which is used below.

Definition. Let λ_j be the roots of the characteristic polynomial

$$L(1,\lambda) = a_0\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0.$$

We assume them to be complex, $\lambda_j \in C$, i.e., Eq. (1) is elliptic. The elliptic equation (1) is called properly elliptic if the roots λ_j are situated equally in the positive and negative imaginary planes:

$$\operatorname{Im}\lambda_j > 0, \ j = 1,2, \ \operatorname{Im}\lambda_k < 0, \ k = 3, \ 4.$$

Assume that all roots of the characteristic equation $a_0\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4 = 0$ are simple and not equal to $\pm i$ and Eq (1) is improperly elliptic, that is one of the next conditions are fulfied :

$$\operatorname{Im}\lambda_1 > 0, \quad \operatorname{Im}\lambda_2 > 0, \quad \operatorname{Im}\lambda_3 > 0, \quad \operatorname{Im}\lambda_4 < 0, \quad (3)$$

$$\operatorname{Im}\lambda_1 > 0, \quad \operatorname{Im}\lambda_2 > 0, \quad \operatorname{Im}\lambda_3 > 0, \quad \operatorname{Im}\lambda_4 > 0. \quad (4)$$

3. Neumann problem in complex plane

We consider here Neumann problem (2) for improperly elliptic class (3). Let us pose $x = \frac{z+\overline{z}}{2}$, $y = \frac{z-\overline{z}}{2i}$, then $\frac{\partial}{\partial z} = \frac{1}{2}\left(\frac{\partial}{\partial x} - i\frac{\partial}{\partial y}\right)$, $\frac{\partial}{\partial \overline{z}} = \frac{1}{2}\left(\frac{\partial}{\partial x} + i\frac{\partial}{\partial y}\right)$, and $\frac{\partial}{\partial x} = \frac{\partial}{\partial z} + \frac{\partial}{\partial \overline{z}}$, $\frac{\partial}{\partial y} = i\left(\frac{\partial}{\partial z} - \frac{\partial}{\partial \overline{z}}\right)$. Applying these expressions to the derivatives that enter into (1), we obtain the new form of this equation:

$$\left(\frac{\partial}{\partial \overline{z}} - \mu_1\frac{\partial}{\partial z}\right)\left(\frac{\partial}{\partial \overline{z}} - \mu_2\frac{\partial}{\partial z}\right)\left(\frac{\partial}{\partial \overline{z}} - \mu_3\frac{\partial}{\partial z}\right)\left(\frac{\partial}{\partial z} - \nu_1\frac{\partial}{\partial \overline{z}}\right)u(z) = 0, \quad (5)$$

where

$$\mu_1 = -\frac{\lambda_1 - i}{\lambda_1 + i};\ \mu_2 = -\frac{\lambda_2 - i}{\lambda_2 + i};\ \mu_3 = -\frac{\lambda_3 - i}{\lambda_3 + i};\ \nu_1 = -\frac{\lambda_4 + i}{\lambda_4 - i}. \tag{6}$$

We also rewrite the Neumann boundary conditions (2) in complex variables. For this purpose, we use the relations

$$\left.\frac{\partial u}{\partial n}\right|_{\partial K} = z\frac{\partial u}{\partial z} + \bar{z}\frac{\partial u}{\partial \bar{z}},\ \left.\frac{\partial^m u}{\partial n^m}\right|_{\partial K} = \left(z\frac{\partial u}{\partial z} + \bar{z}\frac{\partial u}{\partial \bar{z}}\right)^m,\ m = 2,3.$$

Finally, we have the following boundary conditions:

$$\left. z^2\frac{\partial^3 u}{\partial z^3} + 2z\bar{z}\frac{\partial^3 u}{\partial z^2 \partial \bar{z}} + \bar{z}^2\frac{\partial^3 u}{\partial z \partial \bar{z}^2} + 2z\frac{\partial^2 u}{\partial z^2} + 2\bar{z}\frac{\partial^2 u}{\partial z \partial \bar{z}}\right|_{\partial K} = F_1(z), \tag{7}$$

$$\left. \bar{z}^2\frac{\partial^3 u}{\partial \bar{z}^3} + 2z\bar{z}\frac{\partial^3 u}{\partial z \partial \bar{z}^2} + z^2\frac{\partial^3 u}{\partial z^2 \partial \bar{z}} + 2\bar{z}\frac{\partial^2 u}{\partial \bar{z}^2} + 2z\frac{\partial^2 u}{\partial z \partial \bar{z}}\right|_{\partial K} = F_2(z). \tag{8}$$

Here $F_1(z), F_2(z)$ are functions constructed with the help of specified functions $f_1 \in C^{1,\alpha}(\partial K), f_2 \in C^{\alpha}(\partial K)$ from the Neumann conditions (2).

4. Solvability of the problem (5), (7), (8)

Note that the condition of improper ellipticity of Eq. (1) leads to the conditions

$|\mu_k| < 1, |\nu_k| < 1, k = 1,2.$ Obviously, the solution u(z) of Eq. (5) will have the following form:

$$u(z) = \Phi_1(z + \mu_1\bar{z}) + \Phi_2(z + \mu_2\bar{z}) + \Phi_3(z + \mu_3\bar{z}) + \Psi_1(\bar{z} + \nu_1 z), \tag{9}$$

where $\Phi_l,\ \Psi_l, l = 1,2,3$ – are certain functions fairly smooth and analytic together with their derivatives. We find them from the boundary conditions (7), (8). The principal role in this study will belong to lemma on the representation of analytic functions, arising in first in [11, 12] . We substitute the solution u(z) of the form (9) in (7) and (8). For this purpose, we find:

$$\frac{\partial^3 u}{\partial z^3} = \Phi_1'''(z + \mu_1\bar{z}) + \Phi_2'''(z + \mu_2\bar{z}) + \Phi_3'''(z + \mu_3\bar{z}) + \nu_1^3\Psi_1'''(\bar{z} + \nu_1 z),$$

$$\frac{\partial^3 u}{\partial z^2 \partial \bar{z}} = \mu_1\Phi_1'''(z + \mu_1\bar{z}) + \mu_2\Phi_2'''(z + \mu_2\bar{z}) + \mu_3\Phi_3'''(z + \mu_3\bar{z}) + \nu_1^2\Psi_1'''(\bar{z} + \nu_2 z),$$

$$\frac{\partial^3 u}{\partial z \partial \bar{z}^2} = \mu_1^2\Phi_1'''(z + \mu_1\bar{z}) + \mu_2^2\Phi_2'''(z + \mu_2\bar{z}) + \mu_3^2\Phi_3'''(z + \mu_3\bar{z}) + \nu_1\Psi_1'''(\bar{z} + \nu_1 z),$$

$$\frac{\partial^2 u}{\partial z^2} = \Phi_1''(z + \mu_1\bar{z}) + \Phi_2''(z + \mu_2\bar{z}) + \Phi_3''(z + \mu_3\bar{z}) + \nu_1^2\Psi_1''(\bar{z} + \nu_1 z),$$

$$\frac{\partial^2 u}{\partial z \partial \bar{z}} = \mu_1\Phi_1''(z + \mu_1\bar{z}) + \mu_2\Phi_2''(z + \mu_2\bar{z}) + \mu_3\Phi_3''(z + \mu_3\bar{z}) + \nu_1\Psi_1''(\bar{z} + \nu_1 z),$$

$$\frac{\partial^3 u}{\partial \bar{z}^3} = \mu_1^3\Phi_1'''(z + \mu_1\bar{z}) + \mu_2^3\Phi_2'''(z + \mu_2\bar{z}) + \mu_3^3\Phi_3'''(z + \mu_3\bar{z}) + \Psi_1'''(\bar{z} + \nu_1 z),$$

$$\frac{\partial^2 u}{\partial \bar{z}^2} = \mu_1^2\Phi_1''(z + \mu_1\bar{z}) + \mu_2^2\Phi_2''(z + \mu_2\bar{z}) + \mu_3^2\Phi_3''(z + \mu_3\bar{z}) + \Psi_1''(\bar{z} + \nu_1 z).$$

Using methodology which has been proposed in the work [6] we arrive to the following equation for unknown coefficients of functions $\Phi_l,\ \Psi_l,\ l = 1,2,3.$

$$\Delta_k\left(M_{k-1} + 2\mu_1\mu_2\mu_3\nu_1 M_{k+1} + \mu_1^2\mu_2^2\mu_3^2\nu_1^2 M_{k+3}\right) = S_k. \tag{10}$$

Here $M_k = \left(A_k; B_k; C_k; D_k\right)^T$, $S_k = \begin{pmatrix} F_k \\ F_{-k} \\ G_k \\ G_{-k} \end{pmatrix}$, k=1,2,….are coefficients of functions Φ_l, Ψ_l, $l = 1,2,3$ and

$$\Delta_k = \begin{pmatrix} 1 & 1 & 1 & \nu_1^{k+1} \\ \mu_1 & \mu_2 & \mu_3 & \nu_1^k \\ \mu_1^k & \mu_2^k & \mu_3^k & \nu_1 \\ \mu_1^{k+1} & \mu_2^{k+1} & \mu_3^{k+1} & 1 \end{pmatrix}.$$

Thus, the sufficient condition of solvability of Eq. (10) and, hence, the Neumann problem (1), (2) is

$$\det \Delta_k \neq 0, \ \forall k. \tag{11}$$

Note that a similar condition arose in paper [7], where conditions of the existence and uniqueness of solution of the Dirichlet problem for improperly elliptic Eq. (1) with condition (3) were investigated. The solution of (10) exists because, according to (11), there exists Δ_k^{-1}. Performing the substitution

$$\widetilde{S}_k = \Delta_k^{-1} \cdot S_k, \ \mu_1 \mu_2 \nu_1 \nu_2 = \delta. \tag{12}$$

we rewrite Eq. (10) in the form

$$M_{k-1} + 2\delta M_{k+1} + \delta^2 M_{k+3} = \widetilde{S}_k. \tag{13}$$

The obtained vector equation (13) is a fourth-order recursive equation with constant coefficients (see [8]). Its solution M_k , according to the Moivre formulas [8], has the form

$$M_k = Q_1(k-1)\left(\frac{i}{\sqrt{\delta}}\right)^k + k \cdot Q_2(k-1)\left(\frac{i}{\sqrt{\delta}}\right)^k + Q_3(k-1)\left(-\frac{i}{\sqrt{\delta}}\right)^k + k \cdot Q_4(k-1)\left(-\frac{i}{\sqrt{\delta}}\right)^k, \tag{14}$$

with some polynomials $Q_j(k-1)$, $j = 1,\ldots,4$.

5. Main result

Such a way from the previous sections we are coming to the theorem on solvability the Neumann problem (2) for the class (3) of improperly elliptic fourth-order equations (1).

Theorem. Suppose that Eq. (1) is improperly elliptic, and $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ are roots of corresponding characteristic equation satisfying the conditions (3), and: $\lambda_k \neq \lambda_j, k \neq j$, $\lambda_j \neq \pm i, \ \forall j = 1,\ldots,4$. Let also $\mu_1 = -\dfrac{\lambda_1 - i}{\lambda_1 + i}; \mu_2 = -\dfrac{\lambda_2 - i}{\lambda_2 + i}; \mu_3 = -\dfrac{\lambda_3 - i}{\lambda_3 + i}; \nu_1 = -\dfrac{\lambda_4 + i}{\lambda_4 - i}$ satisfy the condition (11):

$$\det \begin{pmatrix} 1 & 1 & 1 & \nu_1^{k+1} \\ \mu_1 & \mu_2 & \mu_3 & \nu_1^k \\ \mu_1^k & \mu_2^k & \mu_3^k & \nu_1 \\ \mu_1^{k+1} & \mu_2^{k+1} & \mu_3^{k+1} & 1 \end{pmatrix} \neq 0,$$

and functions $f_1 \in C^{1,\alpha}(\partial K), f_2 \in C^{\alpha}(\partial K)$, $0 < \alpha < 1$ from (2) are specified on the boundary ∂K and can be prolonged to analytic functions in the disk K and outside it. Then the

Neumann problem (1), (2) will have a solution in the space $C^4(K)\cap C^{3,\alpha}(\overline{K})$, which can be represented as

$$u(z)=\Phi_1(z+\mu_1\bar{z})+\Phi_2(z+\mu_2\bar{z})+\Phi_3(z+\mu_3\bar{z})+\Psi_1(\bar{z}+\nu_1 z),$$

where Φ_l, Ψ_l, $l=1,2,3$ are certain functions fairly smooth and analytic together with their derivatives and the column vector $M_k=(A_k;B_k;C_k;D_k)^T$ of coefficients A_k,B_k,C_k,D_k, $k=1,2,...$ of the expansion of functions Φ_l, Ψ_l, $l=1,2,3$ is determined by the Moivre formulas (14).

Moreover, conditions (11) hold almost everywhere and the kernel of the Neumann problem (1), (2) is finite- dimensional and has

$$d=\sum_{k=4}^{\infty}(4-rank\,\Delta_k)$$

linearly independent elements, which can be represented by the Chebyshev`s polynomials.

Proof. The first part of theorem has been already proved in sections 2, 3. Let us investigate the kernel of the Neumann problem for improperly elliptic equations under condition (3) and build the elements of kernel with the help of Chebyshev`s polynomials. For this aim fist of all let us study the asymptotic behavior of the determinant of matrix

$$\Delta_k=\begin{pmatrix} 1 & 1 & 1 & \nu_1^{k+1} \\ \mu_1 & \mu_2 & \mu_3 & \nu_1^k \\ \mu_1^k & \mu_2^k & \mu_3^k & \nu_1 \\ \mu_1^{k+1} & \mu_2^{k+1} & \mu_3^{k+1} & 1 \end{pmatrix} \tag{15}$$

for sufficiently large k. Let $|\mu_1|>|\mu_2|>|\mu_3|$ and $\alpha=\dfrac{\mu_2}{\mu_1}$, $\beta=\dfrac{\mu_3}{\mu_1}$, $\gamma=\nu_1\mu_1$. Then we have

$$\det\,\Delta_k\approx\det\begin{pmatrix} 1 & 1 & 1 & \gamma^{k+1} \\ 1 & \alpha & \beta & \gamma^k \\ 1 & \alpha^k & \beta^k & \gamma \\ 1 & \alpha^{k+1} & \beta^{k+1} & 1 \end{pmatrix}\approx(1-\gamma)(\beta-\alpha)\neq 0,$$

The fact that condition (11) does not hold for at least some k (i.e., the kernel is nontrivial) is illustrated by the following example. In (15), let k = 3; and $\mu_1=\dfrac{2}{3}, \mu_2=-\dfrac{1}{2}, \mu_3=0, \nu_1=\dfrac{1}{2}$,

Then det Δ_4 = 0. Thus, for improperly elliptic equations satisfying condition (3), condition (11) does not satisfied only for finitely many k (so, condition (11) is true almost everywhere); consequently, in this case, the kernel of the Neumann problem (1), (2) is finite-dimensional and has

$$d=\sum_{k=4}^{\infty}(4-rank\,\Delta_k).$$

This fact is consequence of analogous result about kernel dimension of corresponding Dirichlet problem for the improperly elliptic fourth equation of class (3) ([7]) and famous fact about equality of dimensions of Dirichlet and Neumann problems.

Now let us build elements of the kernel of Neumann problem. In the paper [7] it has been shown that functions

$$u_n(z)=\sum_{k=1}^{3}C_k\left(\frac{1}{2n}T_n(\mu_k z+\bar{z})-\frac{1}{2(n-2)}T_{n-2}(\mu_k z+\bar{z})\right)+ \\ +C_4\left(\frac{1}{2n}T_n(-z-\nu_1\bar{z})-\frac{1}{2(n-2)}T_{n-2}(-z-\nu_1\bar{z})\right) \quad (16)$$

are elements of kernel of Dirichlet problem for improperly elliptic fourth order equation of class (3): $u_n\big|_{\partial K}=0,\ \frac{\partial u_n}{\partial \vec{n}}\big|_{\partial K}=0.$ Here T_k are Chebyshev`s polynomials.

Let us take $u_n = \frac{\partial^2 v_n}{\partial \vec{n}^2}$, where v_n are elements of kernel of Neumann problem. From the formulae (16), $u_n = \frac{\partial^2 v_n}{\partial \vec{n}^2}$ and $\int T_{n-1}(z)dz=\frac{1}{2n}T_n(z)-\frac{1}{2(n-2)}T_{n-2}(z),$ we have elements v_n of the kernel of Neumann problem for case (3) of improperly elliptic equation (1):

$$v_n(z)=\sum_{k=1}^{3}C_k\Big(\frac{1}{8n(n+1)(n+2)}T_{n+2}(\mu_k z+\bar{z})-\frac{n^2-5n}{8n^2(n^2-1)(n-2)}T_n(\mu_k z+\bar{z})+$$

$$+\frac{n^2-3n+6}{8n(n-2)^2(n-3)(n-1)}T_{n-2}(\mu_k z+\bar{z})-\frac{1}{8(n-2)(n-3)(n-4)}T_{n-4}(\mu_k z+\bar{z})\Big)+$$

$$+C_4\Big(\frac{1}{8n(n+1)(n+2)}T_{n+2}(-z-\nu_1\bar{z})-\frac{n^2-5n}{8n^2(n^2-1)(n-2)}T_n(-z-\nu_1\bar{z})+$$

$$+\frac{n^2-3n+6}{8n(n-2)^2(n-3)(n-1)}T_{n-2}(-z-\nu_1\bar{z})-\frac{1}{8(n-2)(n-3)(n-4)}T_{n-4}(-z-\nu_1\bar{z})\Big).$$

So, theorem is proved.

Conclusions

In the present paper it has been investigated the Neumann problem in a disk for one class of improperly elliptic fourth order differential equation in general case, that is roots $\lambda_1,\lambda_2,\lambda_3,\lambda_4$ are different, do not equal $\pm i$ and satisfy conditions:

$$\operatorname{Im}\lambda_1>0,\quad \operatorname{Im}\lambda_2>0,\quad \operatorname{Im}\lambda_3>0,\quad \operatorname{Im}\lambda_4<0.$$

We expanded the methods of investigations of our previous works [6, 7] , which were applied for studying the properly elliptic Neumann problem for the case of improperly elliptic Neumann problem, obtained sufficient condition for existence of solution of posed Neumann problem in space $C^4(K)\cap C^{3,\alpha}(\overline{K}), 0<\alpha<1,$ and constructed solution of such problem with the help of Chebyshev`s polynomials.

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Анотація. ***К.О. Буряченко. Розв'язність задачі Неймана для неправильно еліптичних рівнянь четвертого порядку.*** *В роботі досліджено та встановлено достатні умови розв'язності задачі Неймана для одного класу неправильно еліптичних рівнянь четвертого порядку загального вигляду в крузі K. Доведено існування розв'язку задачі в просторі* $C^4(K)\cap C^{3,\alpha}(\overline{K})$. *За допомогою поліномів Чебишева побудований розв'язок задачі в явному вигляді.*

Ключові слова: неправильно еліптичні рівняння, правильно еліптичні рівняння, диференціальні рівняння в частинних похідних четвертого порядку, задача Неймана, задача Діріхле, ядро, поліноми Чебишева.

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